Determination of the behaviour of a sedimentary rock mass: comparison of measured static and dynamic properties.

Determination du comportement d'un massif rocheux sedimentaire : comparaison entre mesures statiques et dynamiques

Bestimmung des geotechnischen verhaltens der felsmasse

Gegenuberstellung festgestellter statischer und dynamischer werte

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ABSTRACT: The assignment of the elastic modulus of a rock mass under "static" loading conditions is traditionally measured by in-situ tests such as the plate loading test. In the present work this methodology is used in two different ways: in the first approach the rock mass in question is considered as a linear, elastic, homogeneous and isotropic medium, characterized by a "smoothed" uniform value of the modulus of elasticity, whereas in the second approach the same rock mass is treated as a non homogeneous medium exhibiting a modulus of elasticity varying with depth. The next part of the study is dedicated to the determination of the dynamic properties of the same rock mass by measurements of P and S waves velocities. A comparison of the elastic properties of the examined rock mass as they have been calculated through "dynamic" and "static" approaches, reveals considerable differences which are qualitatively explained. The applicability of such a multi-way approach for the estimation of the elastic parameter is very helpful for large scale engineering projects such as a dam, where the correct choice of the design parameters for the anticipated behaviour of the foundation is important.

RÉSUMÉ: La determination du module de l' elasticite d' une masse rocheuse se fait par tradition par des essais sur place, tel que l'essai de plaque. Dans le cadre de cette etude, la masse rocheuse est approchee des deux manieres differentes: en premier lieu, la masse rocheuse est consideree comme lineaire, elastique, homogene et isotrope, decrite par un module d'elasticite moyen et uniforme, alors que, en deuxieme lieu la meme masse rocheuse s'avere non homogene avec un module d'elasticite fonction de la profondeur. La deuxieme partie du travail est consacree a la mesure des proprietes dynamiques de la masse rocheuse. La comparaison entre les valeurs du module elastique mesure par des methodes statiques et dynamiques, au moyen des mesures des vitesses des ondes P et S, revele des differences considerables dont l'explication est tentee qualitativement. L' utilite de la mesure des proprietes elastiques de maniere multiple et alternative est surtout demontree dans le cas des projets a echelle importante, tels que le cas d'un barrage l'est, ou le choix correct des parametres utilises pour le dimensionnement des fondations joue un role important.

ZUSAMMENFASSUNG: : Die Bestimmung des Elastizitätsmoduls unter statischer Belastung , mit örtlichen Prüfungen wird traditionel mit der Methode der Plattenbelastung durchgeführt.In dieser Arbeit wird die Untersuchung mit zwei verschiedenen Verfahren durchgefuhrt : Die erste Untersuchung der zu prüfenden Felsmasse setzt voraus linearen, elastischen, isotropen und homogenen Halbraum , bei diesem ist charakteristisch ein mittelmäsiger konstanter Wert des Elastizitätsmoduls , während im zweiten Fall die gleiche Felsmasse zeigt sich inhomogen mit veränderlichem Elastizitätsmoduls in Bezieung mit der Tiefe. Der nächste Teil der Arbeit behandelt die Bestimmung dynamischer Werte der gleichen Felsmasse unter Geschwindigkeitsfortpflanzung der P und S Wellen. Die Beziehung zwischen statischer und dynamischer Werte , führt zu nutzlichen Bemerkungen , die qualitativ ausgewertet werden. Zusammenfassend : Die Nutzung der Ergebniswerte der elastischen Beziehungen mit verschiedenen Methonen , hat grosse Bedeutung für grössere Bauten des Bauingenieurs , wie der Bau von Talsperren bei denen die richtige Auswahl der charakteristischen Werte von Bedeutung ist.

1 INTRODUCTION

The present study refers to a site in the Northeastern part of Greece where a dam is scheduled to be constructed. One important issue, among others, is the correct and accurate evaluation of the elastic properties of the rock mass where the dam is founded. From the geological point of view, the examined foundation area consists of sandstones and siltstones, while locally tuffs may appear. The in situ plate loading tests have been performed on both right and left abutment, in the interior of two investigative galleries, respectively named AD-R (length of 50m, right abutment) and AD-L (length of 100m, left abutment). The right abutment exhibit a very steep slope consisting of unweathered to slightly weathered and slightly fragmented, grey sandstones with few sets of discontinuities practically closed (<1mm) and ash-green coloured siltstones also slightly weathered and poorly fragmented. The left abutment has a very smooth slope and consists of sandstones, siltstones, tuffs and laminated silty marls. The two former formations are slightly weathered and are considered as the "strong" part of the rock formations, while the latter two are the rather "weak" part and they are presented weathered, fragmented and significantly less structured than the former. Three loading plate tests have been realized in the gallery AD-L and one in the AD-R. A geophysical refraction survey has also been developed on the smooth slope of the left abutment in order to deduce the dynamic properties of the underground geomaterials that would condition the foundation behaviour.

2 ASSESSMENT OF THE ROCK MASS ELASTIC PROPERTIES UNDER STATIC LOADING

The in situ plate loading tests under static conditions are performed by increasing and decreasing steps of normal loading (cycles of loading – unloading) measured by a digital pressure transducer, while the induced vertical deformations (elastic or elastoplastic) of the loaded rock mass can be measured in two different ways described hereafter, leading per case to divergent results. This discrepancy is commented in a latter stage of this communication:

a. the vertical induced displacements due to normal loading of the plate are directly measured by means of accurate digital indicators suitably placed on the plate and the adjacent rock mass. This methodology, traditionally used in practice, leads to the determination of "smoothed" uniform elastic moduli with the assumption that the tested rock mass behaves as a linear, elastic, homogeneous and isotropic halfspace,

b. the vertical deformations of the examined rock mass are directly measured via a moving extensometer placed in a central hole in the interior of the loaded area. The imposed axial deformations due to normal loading are recorded with strain gages properly adjusted on the moving extensometer at different depths along the rock mass interior. This methodology, less commonly used, is more appropriate to describe a non-homogeneous behaviour of the rock mass that is often a more realistic assumption.

2.1 *Methodologies to calculate the modulus of elasticity under static loading*

The most widely used methodology to estimate the modulus of elasticity of a rock mass is the one based on the assumption of homogeneous, isotropic and elastic halfspace. In this case a uniform distribution of the normal loading stress on the interface between the plate and the tested rock mass (i.e. "flexible" plate) is also assumed. This methodology is based on the following equation:

$$\Delta z = \Delta\sigma[(1+\nu)r((zA/r)+(1-\nu)H] / E \qquad (1)$$

therefore : $E = I(\Delta\sigma/\Delta z)$ where,

$\Delta\sigma$: uniform normal stress developed on the interface of the plate and the tested rockmass

Δz : vertical displacement as measured by the surface dial gages

I : coefficient dependent on the depth of the measuring point (z), the radius of the loaded area (r=0.25m), the Poisson's ratio (ν=0.18) and the values of the functionals A, H calculated from relevant tables (Foster & Ahlvin, 1954).

Another alternative methodology to calculate the "static" modulus of elasticity is realized via the moving extensometer positioned in a central hole under the loaded rock mass area while its edge in the present study reaches a depth of about 2.40m. This methodology is also based on the assumption of a linear, elastic, isotropic halfspace and a uniform normal stress distribution on the plate – rock mass interface (Poulos & Davis 1974). Therefore, it is possible to register differentiated moduli of elasticity versus depth and hence non-homogeneous behaviour of tested media such as a rock mass. The mathematical expression of the aforementioned approach is given as follows:

$$E = \Delta(\sigma_z - 2\nu\sigma_r) / \Delta\varepsilon_z \qquad (2)$$

where :

σ_z : normal stress at a depth z under the central point of the loaded area. For a surface normal load $\sigma_v=\sigma$, then $\sigma_z=I_z\sigma$, where $I_z = 1-[1/(1+(r/z)^2]^{3/2}$, r=0.25m

ν : Poisson ratio (ν=0.18)

σ_r : radial (horizontal) stress in the middle of the separated zones at a depth z due to a surface normal stress $\sigma_v=\sigma$, is $\sigma_r=I_r\sigma$, $I_r=0.5\{(1+2\nu)-[2(1+\nu)z/(r^2+z^2)^{1/2}]+[z^3/(r^2+z^2)^{3/2}]\}$

$\Delta\varepsilon_z$: variation of the vertical deformation at a certain depth z corresponding to the stress modification $\Delta(\sigma_z-2\nu\sigma_r)$.

2.2 *In situ plate loading results*

Four plate loading tests have been performed in the framework of this study. One of them (PL-1) is situated in the investigative gallery excavated on the right abutment of the river (AD-R), while the rest three of them are positioned in the interior of the investigative gallery of the left abutment (AD-L) with an overburden of about 17m for plate loading tests named PL-2 and PL-3, and 14m for the test PL-4. The two different approaches to estimate the "static" modulus of elasticity give mean values for each location of the test which are presented in Table 1. Given that the rock mass behaves in all tested sites as an elastoplastic material, the "static" modulus of elasticity is calculated on the reloading branch after a first cycle of loading–unloading has

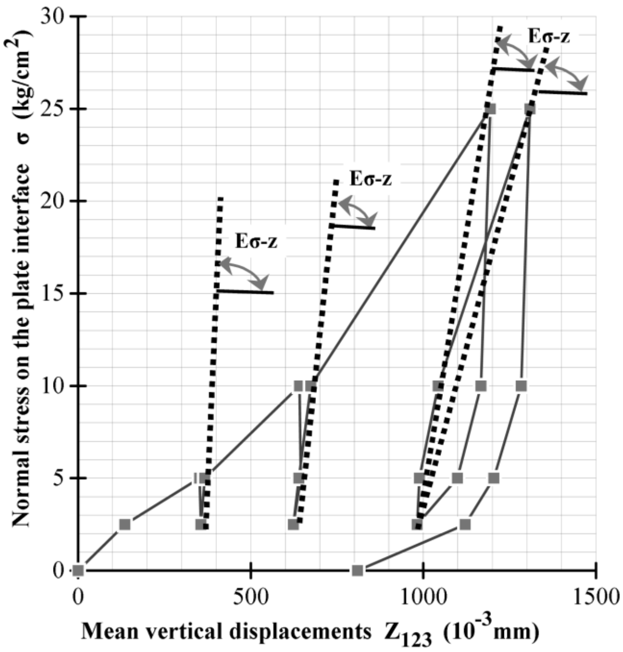

Figure 1. Typical presentation of normal stress loading-unloading cycles of the plate loading test. Dashed lines indicate the estimation of the modulus of elasticity along the re-loading branches.

been performed in order to exclude the influence of the permanent deformations of the tested rock mass as shown in Figure 1.

Commenting the mean values presented in Table 1, it is observed that the values of the "static" modulus of elasticity measured by the surface dial gages (1st line of Table 1) are rather high with a limited dispersion, corresponding to a good quality of a rock mass as this was already shown by the existing geotechnical boreholes and the RMR classification. These values ($E_{\sigma\text{-}z}$) represent the mean global elastic characteristics of the tested rock mass irrespective of the examined depth (assumption of homogeneous medium).

Table 1. Summarized mean values of the "static" moduli of elasticity

Mean values E	PL-1 (AD-R)	PL-2 (AD-L)	PL-3 (AD-L)	PL-4 (AD-L)
Mean $E_{\sigma\text{-}z}$ (MPa) Surface dial gages	5590	3725	5080	6200
Mean $E_{\sigma\text{-}\varepsilon}$ (MPa) Zone 1 (0 – 29.2 cm)	3280	795	/	/
Mean $E_{\sigma\text{-}\varepsilon}$ (MPa) Zone 2 (29.2 – 57.0 cm)	4640	790	2705	1150
Mean $E_{\sigma\text{-}\varepsilon}$ (MPa) Zone 3 (57.0 – 88.4 cm)	3155	910	1680	635
Mean $E_{\sigma\text{-}\varepsilon}$ (MPa) Zone 4 (88.4 – 232.2 cm)	/	2900	/	1010

On the other hand, the values of the mean elastic moduli for each zone of depth ($E_{\sigma\text{-}\varepsilon}$) result from direct measurements of the vertical deformations of a central point of the loaded rock mass via the moving extensometer and they are lower or equal to the former. Those differences are rather small in the case of PL-1 and PL-3, while they are more pronounced in the case of the sites PL-2 and PL-4. This discrepancy might be attributed to the quality of the rock mass. In fact, PL-1 and PL-3 belong to a better quality of rock mass compared to the one of the sites PL-2 and PL-4, therefore, the variation of the rock mass properties with the depth is less pronounced for those sites. Consequently, the agreement of the calculated values for the "static" modulus of elasticity irrespective of the adopted methodology, is most probably due to the fact that the simulation of the rock mass with a homogeneous, isotropic and elastic halfspace is relatively sound under those circumstances. For the case of a typically non homogeneous medium such as the rock mass of the sites PL-2 and PL-4, where shear zones interfere locally or intensive local weathering may appear, the final results depend upon the methodology adopted for the measurement of the deformations or the displacements. In other words, the discontinuous and non-homogeneous character of such a rock mass is promoted and therefore the assumption of a linear, elastic, homogeneous, isotropic and continuous medium is argued. In particular, the existing stratification or schistocity alters noticeably the distribution of the normal stress imposed at the surface of the rock mass. In this case the stress distribution is different and the calculated values of the deduced elastic moduli might be misleading (Gaziev & Erlikhman 1971).

3 DETERMINATION OF DYNAMIC PROPERTIES BY GEOPHYSICAL REFRACTION SURVEY

A geophysical refraction survey was conducted along the smooth slope of the largest left abutment in order to deduce the dynamic properties of the underground geomaterials. The survey included five refraction profiles (S1 to S5), along which both P and S wave measurements were performed. The main refraction line (S1) was performed along the proposed dam axis and consisted of 3 spreads of 24 recording geophones (S11, S12, S13). All other lines were single-spread and have been oriented perpendicular to this main line in order to have a better control along the main refraction line (S1).

For each spread, recording was performed for waves generated at three points (sources), which were located at the beginning, middle-point and end of each spread. In order to improve the signal-to-noise ratio, waveform stacking was performed. For the generation of P-waves a dropping weight (55kg dropped from 3m) was used. S-waves were generated by dropping the same weight on an asymmetric wedge. After stacking several waveforms, the wedge was reversed (turned 180^{o}) and the waveform stacking polarity was also reversed on the recorder. This reversal ensured the cancellation of the contribution of the P-waves, whereas the asymmetric shape of the wedge resulted in the stacking of the S-wave energy.

Before the final interpretation, static corrections were applied to the data in order to correct for the offset of certain geophones from each refraction profile. The processing of the obtained results was performed using the SIP SHELL software of Rimrock Geophysics. The interpretation method employed by this software is the method of Scott (1973), which is a combination of the classic Plus-Minus and Time-Delay methods. Details about the method and comparison with other techniques can be found in Scott & Markiewich (1990).

The obtained results along the main refraction line-dam axis, S1, are shown in the top part of Figure 2. Three formations (denoted by A, B, and C) have been recognized and are presented together with their average P and S velocities. In the same figure, the position of the 3 spreads of this profile (S11, S12, S13) and the intersecting geophysical lines (S2 to S5), as well as the position of the geophones and sources are also shown. In general, the propagation velocities within each formation (both P and S) showed little variance (typically less than 20%), which implies a rather smooth horizontal spatial variation of the corresponding "dynamic" moduli.

In the bottom part of Figure 2 the corresponding cross-section based on the geomechanics classification (RMR system) as determined from the geotechnical boreholes and surface survey is also shown. The position of the 6 boreholes is also shown in the same figure. Four formations (GZ-1 to GZ-4), with different RQD and crack density have been recognized and are presented in the cross-section along with the alluvial deposits (Al). A very good geometrical correlation is recognized between the results of the geophysical and geotechnical survey. Geophysical formations A, B, and C correspond almost identically to the geotechnical classes GZ-4, GZ-2 and GZ-1. Formation A also includes the alluvial deposits (Al). The small differences in the geometrical characteristics of the various formations/classes between the two sections could be attributed to the small difference between the spatial position of the two cross-sections and the interpolation of the borehole results. The correlation between the results of the geophysical survey and the geotechnical classification is further demonstrated by the correlation of the average RQD and the P-S velocity values, shown in the top-right inset plot of Figure 2.

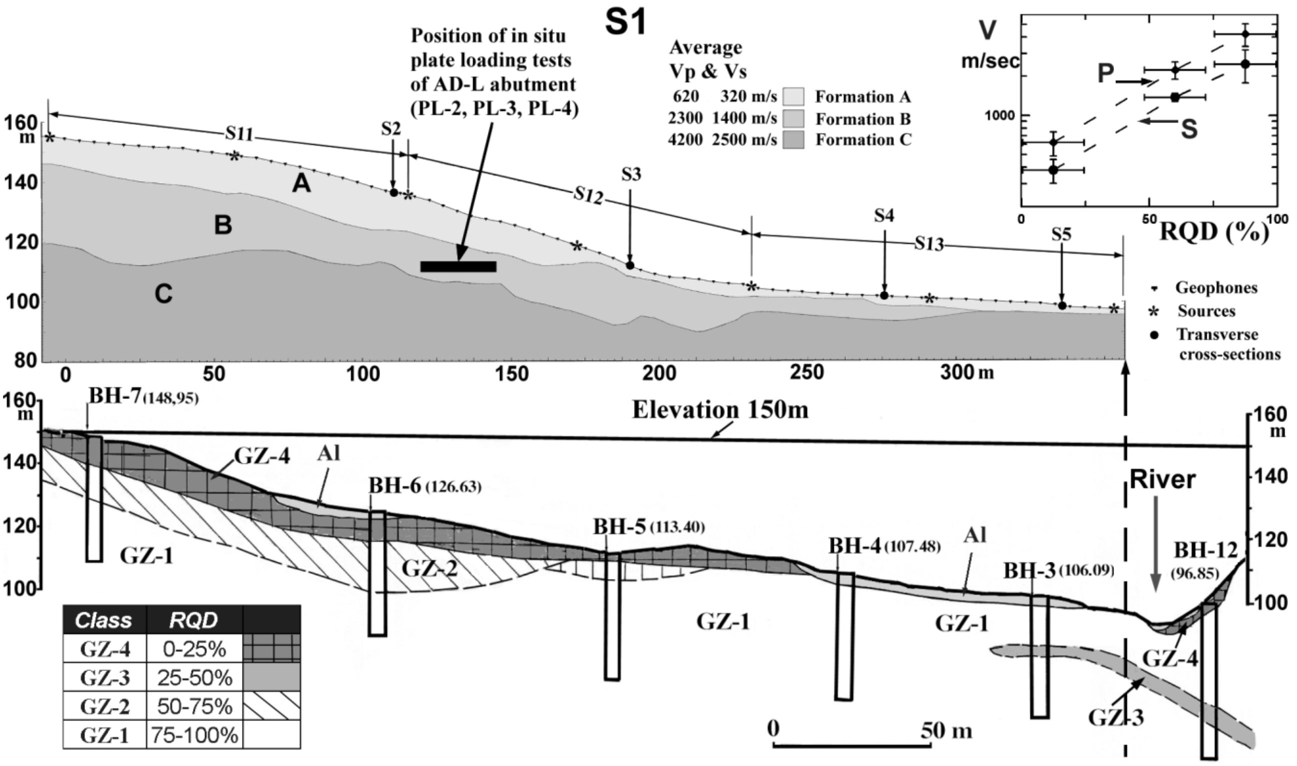

Figure 2. Geophysical (top) and geotechnical (bottom) cross-section along the left abutment of the dam axis. Notice the very good agreement of the geometrical features of the two cross-sections, as well as the correlation of the average RQD values with the P and S velocities (top-right inset figure).

The variance of the RQD and the body-wave velocities for each formation is also shown in the same plot. A good correlation is observed for both the P and S velocities with the average RQD values, verifying thus the validity of the proposed geophysical-geotechnical correlation.

In order to obtain the "dynamic" moduli for the previously determined formations, the density, ρ, of each formation was measured from selected samples of the borehole survey. Using these densities, the "dynamic" Poisson ratio, shear and Young moduli were estimated using the following relations:

$$G_{dyn} = \rho\, V_S^2$$

$$E_{dyn} = \rho\, V_S^2\, (3V_P^2 - 4V_S^2) / (V_P^2 - V_S^2) \qquad (3)$$

$$\nu = (V_P^2 - 2V_S^2) / (V_P^2 - V_S^2)$$

Using equations (3) the dynamic properties are obtained and presented in Table 2.

Table 2. Dynamic properties of the identified geophysical formations

Formation	Geomechanical class	G_{dyn} MPa	Edyn MPa
A	GZ-4 / Al	312±135	747±240
B	GZ-2	4950±720	12200±1700
C	GZ-1	16700±9600	41300±16600

The Poisson ratio, ν, is poorly controlled by results obtained from geophysical prospecting since small errors in the estimation of V_P and V_S introduce large deviations in ν. In general, values in the range 0.20-0.24 were obtained for the Poisson's ratio.

The variance of the dynamic properties presented in Table 2 is due to the gradual decrease of the "dynamic" moduli within each formation as we move towards the river (right part of Figure 2). This is more prominent for formation C (GZ-1) where E_{dyn} drops from approximately 60000 MPa in the left part of line S1 (average depth 50m) to 17000 MPa close to the river (average depth 15m). On the other hand, formation B (GZ-2) shows a smaller variability of E_{dyn}. For formation A, areas which correspond to GZ-4 exhibit E_{dyn} values in the range of 700 to1100 MPa, whereas the alluvial deposits show even smaller average values (less than 650 MPa).

4 DISCUSSION - CONCLUSIONS

The determination of the elastic characteristics of the examined rock mass via the plate loading test and the geophysical refraction survey have revealed some differences. A discussion concerning possible explanations and comparisons between the obtained results is undertaken hereafter, bearing in mind that remarks refer only to the left abutment and tests performed in gallery AD-L since "dynamic" measurements exist only for the above abutment.

The "static" measurements ($E_{\sigma\text{-}z}$) coming from surface vertical displacements via digital dial gages placed on the loaded plate and the surrounding rock surface, provide constant values of the modulus of elasticity ranging from 3725 to 6200 MPa for formation B. Those values consider the loaded rock mass as a homogeneous, linear, elastic and isotropic medium whose behaviour is globally evaluated by an overall constant value. The alternative way of estimating the "static" modulus of elasticity ($E_{\sigma\text{-}\varepsilon}$) is based on the direct measurement of the axial deformations of the central part of the loaded rock mass to a depth of 2.40m. This approach, based on the moving extensometer, divides the loaded subsurface in four arbitrary zones of unequal depth and gives respectively four different values of the modulus of elasticity dependent on the local conditions included in each zone. Therefore, the calculated moduli might exhibit considerable differences due to the rock mass inhomogeneity among other reasons. However, it is interesting to note that values of $E_{\sigma\text{-}\varepsilon}$ concerning the same formation B of the left abutment, range from 635 to 2900 MPa and they are clearly lower to those measured with the first approach ($E_{\sigma\text{-}z}$). A possible explanation of the observed discrepancy could be attributed to the stress distribution severely affected by the anisotropy of the formations, which results in a normal stress distribution considerably different to the one assumed by Boussinesq for an isotropic medium. For a case of a vertical set of discontinuities or schistocity, the stress distribution under the central point of the interface between the loading plate and the tested rock mass could be almost 2.5 times higher than the one proposed for an isotropic medium. In cases where vertical induced anisotropy is pronounced, the overall estimation of a mean value of the modulus of elasticity $E_{\sigma\text{-}z}$ seems more appropriate to describe the rock mass behaviour for the following reasons:

a. the points of surface measurement on the loading plate are radially spaced at a distance x=0.845r (r=0.24m, radius of the plate), where the normal stress at the interface is the same, irrespective of the plate behaviour as a "rigid" or "flexible" medium,

b. for the case of vertical or near vertical anisotropy ($\alpha \simeq 90^0$), as it is the case in some of the examined sites, the stress values at the interface beneath the measuring points is quite simular with the expected values for an isotropic medium (Sasaki et al. 1995). However, the stress distribution of the loaded rock mass beneath the central point of the loading plate and along the central hole of the moving extensometer deviates significantly from the isotropic hypothesis, leading to an underestimation of the modulus of elasticity.

On the other hand, the "dynamic" measurements for formation B exhibit a low variability (11000 to 14500 MPa). For the area (see Figure 2) where the plate loading tests have been performed, the "dynamic" modulus of elasticity, corresponding to a depth of 15 to 20m, was estimated around 11500 MPa. The ratios of this "dynamic" value over the "static" moduli of elasticity result in the following range of values: $E_{dyn}/E_{\sigma\text{-}z}$ = 1.85 – 3.1 and $E_{dyn}/E_{\varepsilon\text{-}z}$ = 5.9 – 10.7. According to the aforementioned comments on the "static" moduli of elasticity, the obtained ratios of 2 to 3 are believed to be more representative for the examined case, given the good quality of the rock mass (formation B).

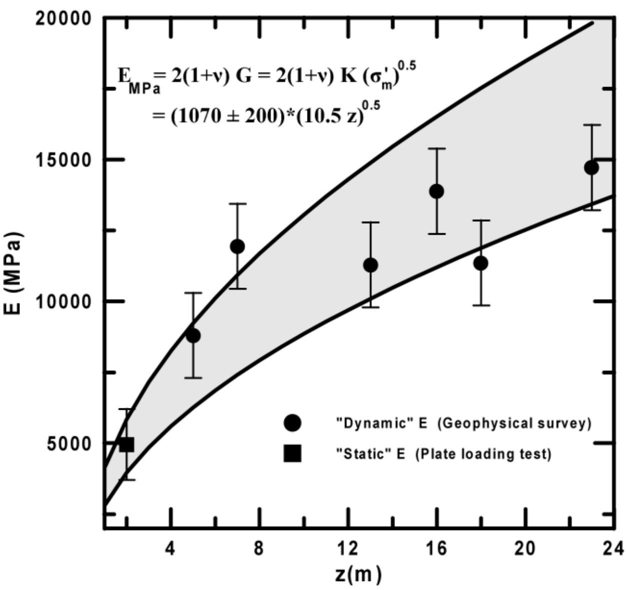

Figure 3. Variation of "dynamic" and "static" E versus depth.

It is interesting to attempt a semi-quantitative explanation of this difference between "static" and "dynamic" estimates. A possible cause of this difference could be the dependence of E on the strain level. Strain levels for the geophysical survey and plate loading tests are respectively of the order of 10^{-6} to10^{-5} and 10^{-4} to10^{-3}. According to the results obtained by Ozkan et al. (1995), the degradation of the shear modulus for a rockfill material used for a dam, begins at the level of shear strains of $2*10^{-4}$ while $G = 0.6G_{dyn}$ for shear strains 10^{-3}. Therefore, for the case of the examined natural rock mass it seems reasonable to assume that for strains ranging from 10^{-4} to 10^{-3} it is rather unlikely to observe important non-linear effects (decrease of E). On the other hand, it is interesting to notice that the values of E_{dyn} depend on the depth or the thickness of the overburden, which is not constant for formation B (Figure 2). This can be observed in Figure 3, where the estimated values of E_{dyn} are plotted against the average depth of formation B. In the same figure, the "static" value, $E_{\sigma\text{-}z} = 5000 \pm 1200$ MPa is also shown, placed at a depth of about 2m (plate loading tests are performed in the interior of an investigative gallery where the normal stress of the overburden is removed). A significant increase is observed with depth for the values of E, which can be attributed to the increase of the mean effective isotropic stress $\sigma'_m=(\sigma'_1+\sigma'_2+\sigma'_3)/3$. In order to quantify this increase of E with depth, the following relation for the shear modulus, G_{dyn}, of cohesionless soils at low strains is used (Hardin & Drnevich 1972, Seed & Idriss 1970):

$$G_{dyn} = K\,(\sigma'_m)^n \quad \text{(MPa)} \tag{4}$$

K is a function of the material and the strain and n is usually assumed to be equal to 0.5. Using the basic law of elasticity:

$$E_{dyn} = 2\,(1+\nu)\,G_{dyn} \tag{5}$$

$$\sigma'_m = (\sigma'_1+\sigma'_2+\sigma'_3)/3 = (\sigma'_1+2\sigma'_3)/3 = (1+2\kappa)\,\sigma'_1/3 \simeq 2\gamma'_R z/3 \tag{6}$$

and assuming a value of v=0.23 for the Poisson ratio (from the geophysical survey and laboratory tests) and an average value of $\gamma_R=26$ kN/m^3 (from borehole samples) resulted in a K value of 435 ± 80, using only the E_{dyn} values, and the following relation for the elastic modulus:

$$E_{dyn} = 2(1+0.23)K(\sigma'_m)^{0.5} = (1070 \pm 200)*(10.5z)^{0.5} \tag{7}$$

The variance of E_{dyn} with depth, z, given by equation (7), as predicted from the measured E_{dyn} values, is also shown in Figure 3 as a grey-shaded area. It results that equation (7) "predicts" quite successfully the measured value of $E_{\sigma\text{-}z}$. Therefore, the "static" and "dynamic" values measured from in situ tests are rather coherent, if the role of the mean effective isotropic normal stress is taken into account. The dependence of the "dynamic" moduli refering to cohesionless soils as a function of the square root of the depth, is quite consistent with the in situ measurements. It also has to be noted that this square root dependence of the elastic modulus, given by equation 4, seems to be rather "strong" for the type of material under study. If the "static" value, $E_{\sigma\text{-}z}$, is also taken into account for the determination of a best-fit relation, (using equations 4, 5, and 6), then equation 7 is modified to:

$$E_{dyn} = (1690 \pm 250)\,(\sigma'_m)^{0.4} \quad \text{(MPa)} \tag{8}$$

As a conclusion, it can be suggested that for the examined rock formation B where the dam is to be founded, a "static" modulus of elasticity of 5000 MPa is realistic for design purposes. Moreover, the corresponding "dynamic" modulus of elasticity of the same rock mass could be deduced from equations (7) or (8) for low strains.

REFERENCES

Foster, C.R. & Ahlvin, R.G. 1954. Stresses and deflections induced by a uniform circular load. *Proc. High Res. Board,* Vol. 33: 467-470.

Gaziev, E.G. & Erlikhman, S.A. 1971. Stresses and strains in anisotropic rock foundation (model studies) *Symposium Soc. Int. Mec. Des Roches, Nancy,* II-1.

Hardin, B.O. & Drnevich, V.P. 1972. Shear modulus and damping in soils: design equations and curves. *Journal of the Soil Mechanics and Foundations Division, ASCE,* 98(SM7): 667-692.

Poulos, H.G. & Davis, E.H. 1974. *Elastic Solution for Soil and Rock Mechanics*. New York: John Wiley & Son.

Sasaki, T. Morikawa, S. Matsukawa, T. & Yoshinaka, R Stability analysis of jointed rock foundations by FEM using the multiple yield models. In Yoshinaka, R. & Kikuchi, K. (eds). *Proceedings of the International Workshop on Rock Foundation, Tokyo, 30 September 1995*: 337-342. Rotterdam: Balkema

Seed, H.B. & Idriss, I.M. 1970. Soil moduli and damping factors for dynamic response analyses. *Report EERC 70-10,* Earthquake Engineering Research Center, University of California, Berkeley.

Scott, J.H. 1973. Seismic refraction modeling by computer: *Geophysics,* 28(2): 271-284.

Scott, J.H. & Markiewicz, R.D. 1990. Dips and chips-PC programs for anlyzing seismic refraction data: *Proceedings, SAGEEP 1990, Golden, Colorado*: 175-200.