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The Applicability of Two-dimensional Inversion Filters in Magnetic Prospecting for Buried Antiquities

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ABSTRACT

Inversion filters can be computed on the basis that a particular signal can be considered as the convolutant of two distinct and analytically determined functions. Therefore, if such filters are convolved with the measured field, they provide the distribution of the function which was not inverted. If the non-inverted function has been chosen in a manner to define the surface projection of the buried target, a plan view of the subsurface conditions has been provided.

The vertical sided prism is justified as the model which resembles the majority of the expected structures in an Archaeological site. Hence, the effect which that produces forms the signal upon which manipulations are done. Several filters are presented which were computed on that basis.

Applications on synthetic and actual data are presented in order to clarify the merits and disadvantages of the technique.

INTRODUCTION

The aim of "archaeological geophysics" is to locate and identify buried ancient relics. Up to date, several techniques have been tested with moderate to high success on this matter.

However, because problems of expropprietation of lands are very commonly met by Archaeologists, more information is needed than a simple pinpointing of localities that might conceale an antiquity. The survey is considered to be succesfully completed if it has as final product a map which more of less gives the plan view of the concealed structures, i.e., the result that would have been drawn if an excavation had taken place. A map in that form aims to help the Archaeologists to decide upon continuation, postpone, alterate or even cancel the progression of workings of civil engineering projects if they are connected with an archaeological site. Of course, the utility of a geophysical product is not exhausted in such cases of "resque excavation" only. The exploitation of such results in designing and better planning any excavation is obvious.

The present study comprises an effort to achieve the presentation of the geophysical maps over buried antiquities in an image form. That image should give an outline of the dimensions of buried structures, should present their centers in exact locations (within the range of accuracy), and of course it should be as reliable as possible.

The problem is attempted to be solved by means of inverse filtration. The convolutional model of the total magnetic field anomalies, which is necessary for this case, was proposed by Bhattacharrya and Navolio (1975) and Bhattacharrya and Chan (1977).

Karousova and Karous (1989) proposed the exploitation of inverse filtration in the geophysical search for antiquities. They constructed the appropriate filters for the vertical cylinder and infinite prism using the equations of Logacev and

Zacharov (1973). Such filters, if convolved with the total field anomaly profiles, produced pronounced and informative results.

Tsokas et al. (1989) attempted to modify the procedure described by karousova and Karous (1989). They used filters which were produced employing the anomaly caused by a rectangular prism model and convolved them with every profile of the geophysical map. In such a way, the most commonly met model was used in a pseudo-two-dimensional procedure. Additionally they used as filter's truncation length the lag where the autocorrelation of the shape function vanishes to zero. Where, shape function is the convolutant to be inverted and thus result into a filter.

The present study is aimed towards a fully two-dimensional procedure. Such a procedure, should be relatively simple and easy applicable, in order to meet the requirements of all kind of excavation projects.

THE CONVOLUTIONAL MODEL

McGrath and Hood (1973) proposed an algorithm to produce the magnetic effect over any prism shaped body. These geometrical models were generated from the thin-plane model by numerical integration. According to that algorithm the following equation was considered (Grant and West, 1965, McGrath and Hood, 1973) which gives the magnetic effect over a thin-plate:

$$\Delta T\ (x,y) = J\ s\ b\ c\ [f(x,y+Y)-f(x,y-Y)], \qquad (1)$$

where, $f(x,y+Y)=T_1 - T_2 - T_3 \bullet (T_4 + T_5) - T_6 \bullet T_7 \qquad (2)$

Figure (1) shows a dipping thin-plane which strikes along Y axis of an orthogonal coordinate system. The symbols used to annotate several quantities have the following explanation:

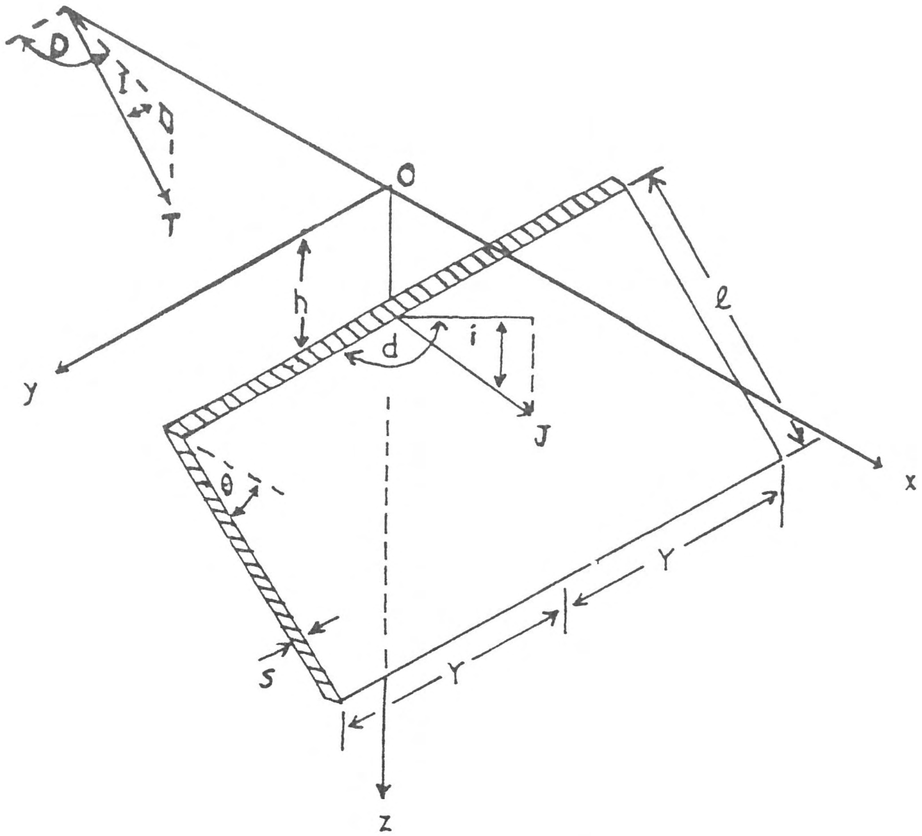

Fig.1. View of the thin-plane model (McGrath and Hood 1973).

J=intensity of Magnetization,

T=Earth's total magnetic field intensity,

s=thickness of the plate,

$b=(\sin^2 i+\cos^2 i \bullet \sin^2 d)^{1/2}=\sin i/\sin\gamma$,

$c=\sin^2 I+\cos^2 I \bullet \sin^2 D)^{1/2}=\sin I/\sin\beta$,

I=angle of inclination of the earth's magnetic field,

i=angle of inclination of the magnetization (-180 to 180 degrees),

d=angle between the horizontal projection of J and the positive Y-axis,

D=angle between the horizontal projection of T and the positive Y-axis,

Y=half-strike length of the plate,

$c^2=(x-l\cos\theta)^2+(h+l\sin\theta)^2$,

θ=angle of dip of plate,

l=width of the plate,

h=depth of the plate,

$E^2 = x^2 + h^2$,

$B = x \cdot \sin\theta + h \cdot \cos\theta$

$A = h \cdot \sin\theta - x \cdot \cos\theta$,

β=angle of inclination of the component of T in the xz plane,

$= \tan^{-1} (\tan I / \sin D)$,

γ=angle of inclination of the component of J in the xz plane,

$= \tan^{-1} (\tan i / \sin d)$.

The quantities at the right hand side of equation (2) are defined as follows:

$$T_1 = \frac{y+Y}{(c^2 + (y+Y)^2)^{1/2}} \cdot \left[\frac{(x - l \cdot \cos\theta)\cos\alpha - (h + l\sin\theta)\sin\alpha}{c^2} \right]$$

$$T_2 = \frac{y+Y}{(E^2 + (y+Y)^2)^{1/2}} \left[\frac{x\cos\alpha - h\sin\alpha}{E^2} \right],$$

$$T_3 = \frac{1}{B^2 + (y+Y)^2} \cdot \left[\frac{A+l}{[c^2 + (y+Y)^2]^{1/2}} - \frac{A}{[E^2 + (y+Y)^2]^{1/2}} \right],$$

$$T_4 = (\cos\alpha \cdot \cos\theta - \cos\beta \cdot \cos\gamma + \cos\beta \cdot \cot D \cdot \cos\gamma \cdot \cot d) \cdot (y+Y),$$

$$T_5 = (\cos\gamma \cdot \cot d \cdot \sin(\theta - \beta) + \cos\beta \cdot \cot D \cdot \sin(\theta - \gamma)) \cdot B,$$

$$T_6 = \cos\gamma \cdot \cot d \cdot \cos(\theta - \beta) + \cos\beta \cdot \cot D \cdot \cos(\theta - \gamma),$$

$$T_7 = \frac{1}{[c^2 + (y+Y)^2]^{1/2}} - \frac{1}{[E^2 + (y+Y)^2]^{1/2}}$$

$\alpha = \beta + \gamma - \theta$.

Let us now consider a model which resembles (or suits to) the majority of structures occurred in archaeological search. Of course, this can be nothing else than a rectangular vertical sided prism. This choise is well justified since the most commonly met features can be represented as a series of such "blocks". For instance, the concealed features are usually building foundation relics, roads, tombs, wall relics, ditches, e.t.c.

Consequently, following the initial algorithm, we consider the vertical sided rectangular prism of figure (2) as been represented by a series of thin plates. On the validity of the superposition principle the magnetic effect which the prism produces can be calculated as the sum of the effects of the individual thin plates. This is given by the following equation:

$$\Delta T(x,y)=J\ s\ b\ c\sum_{i=1}^{N}[f_i(x,y+Y)-f_i(x,y-Y)], \qquad (3)$$

where, N is the number of thin-plates which represent the prism. The expression of total field anomaly is given in equation (3) as a simple summation instead of the elliptical integration used by McGrath and Hood (1973). This is a compromise for the shake of the simplicity of the whole procedure. However, if the number of thin plates used to generate the prism is big enough, the difference is negligible.

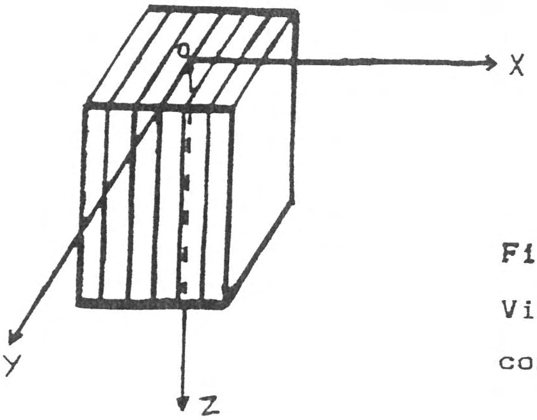

Fig.2.
View of a vertical sided prism composed of a number of thin plates.

Equation (3) can be written as the product of two functions, say D and R. If we define

$$D = J \qquad (4)$$

then, this factor can be reasonably termed as "amplitude function" because it modulates the strength of the anomaly. The function

$$R(x,y) = s\ b\ c \sum_{i=1}^{N} [f_i(x,y+Y)-f_i(x,y-Y)] \qquad (5)$$

can be termed "shape function" since it controls the shape of the anomaly. The shape function must become effectively zero very rapidly as the distance from the body center increases. I.e.

$$R(x,y) \longrightarrow 0 \quad \text{as} \quad x \text{ or } y \longrightarrow \pm\infty \qquad (6)$$

Hence, equation (3) can be presented as

$$\Delta T(x,y) = D \cdot R(x,y) \qquad (7)$$

In order to proceed in a convolutional model the following assumptions must hold:

a)Magnetization of the thin plates is presumably of induced type or, instead, it should be considered along a Known stable direction.

b)Any structure of irregular shape can be simulated by an ensemble of elementary bodies of simple geometric shape. These pieces should be similar in shape and buried at equal depths.

The posetion of the former assumptions is not at expence of the generality. Induced magnetization can be considered as basically responsible for the anomalies over buried relics except pronounced situations (like buried Kilns), where, there is no need for any sort of filtration. On the other hand, the second assumption is self evident in "archaeological geophysics".

Let consider an ensemble of bodies of attitude similar to the one given in figure (2), which are placed at the same depth at points (x ,y) given by

$$x_l = l\Delta_x \qquad l=L_1,\ldots\ldots L_2$$

$$y_m = m\Delta_x \qquad m=M_1,\ldots\ldots,M_2$$

where, Δ_x and Δ_y are the digitization intervals along x and y axis respectively. For the shake of simplicity we can consider that

$$\Delta_x = \Delta_y .$$

The bodies are considered equally magnetized. On the validity of superposition principle the total field anomaly which they produce can be written as:

$$\Delta T(x_i , y_j) = \sum_{\ell=L_1}^{L_2} \sum_{m=M_1}^{M_2} D_{\ell,m} \cdot R(x_i - x_\ell , y_j - y_m) \qquad (8)$$

Equation (8) is an expression of digital two dimensional convolution which becomes more obvious if we rewrite it as:

$$T_{ij} = \sum_{\ell=M_1}^{L_2} \sum_{m=M_1}^{M_2} D_{\ell m} \cdot R_{i-\ell , j-m} \qquad (9)$$

where $\Delta T(x_i , y_j) = T_{ij}$ and $R(x_i - x_l , y_j - y_m) = R_{i-l, j-m}$. The same equation can be now presented in a simpler form as

$$T = D * R \quad , \qquad (10)$$

The spatial distribution of the amplitude function D can be obtained from equation (10) by convolution with the inverse filter R^{-1}. Hence,

$$D = T * R^{-1} \quad , \qquad (11)$$

given that,

$$R * R^{-1} = \theta$$

where, θ, is a unit diagonal matrix.

The minimization of the sum of the square errors $E = R * R^{-1} - \theta$ produces the optimum filter coefficients $\bar{R}^{-1}_{ij}$. Reasonably, $\bar{R}^{-1}_{ij}$ denotes the truncated filter series because inversion of R_{ij} results in an infinitely extending in both directions series (z transform theory).

Such a finite in both directions filter which is optimum in a least square sense, is given by

$$\sum_{j=L_1}^{L_2}\sum_{i=M_1}^{M_2}\overline{R}_{i,j}^{-1}\cdot A_{k-i,l-j}=R_{-k,-l} \tag{12}$$

where,

$$A_{k,l}=\sum_{j=L_1}^{L_2}\sum_{i=M_1}^{M_2}R_{i+k,j+l}\;R_{i,j}$$

is the autocorrelation of the shape function (Kanasewitch 1975).

INVERSION FILTERS

According to the analysis of the previous paragraph, the inversion filters which are produced by equation (12) if convolved with the total magnetic field anomaly should produce two dimensional pulses. Presumably, the subsurface situation which results into the particular magnetic field distribution is composed by structures which can be built up by combinations of the used model.The amplitude of such pulses should be equal to the magnitude of magnetization of the model prism used. Of course, we are not using infinitely long filters and buried bodies are not actually of perfect geometrical form, and burial depth varies. Hence, it is impossible to achieve such a scope in a strictly mathematical manner.

Nevertheless, employing equation (12), several filters were produced. Four of them have been tabulated in the following tables (I) to (IV) and they were annotated as F2, F2A, F3A, F4 respectively. The parameters of the model used to produce the respective filter are also given at the tables. The above filters were selected for demonstration purposes. The difference between the filter F2 and F2A is that the later consists of 7X7 positions against 5X5 positions of the first. The filter F3A was derived using exactly the same model as in the cases F2 and F2A but the prism is located deeper in the F3A model. In all three first cases the model was a with side of 1m. The last filter, F4, was derived using a vertical sided parallilepiped.

The burial depth associated with each model is not the actual depth but the distance between the magnetometer's sensor to the top surface of the model.

TABLE (I)

FILTER F2

PARAMETERS OF THE USED MODEL

Inclination of the Earth's total field:54°

Declination of the Earth's total field with respect to y-axis:90°

Burial depth (depth to the top of the prism):1m

Depth extend:1m

Thickness along N-S direction (X axis):1m

Half strike lenght (y axis):0.5m

Dip of the prism:90°

Number of thin plates used to generate the prism:11

PRODUCED FILTER

0.002	0.006	0.01	0.006	0.002
-0.01	-0.005	0.05	0.005	-0.01
0.06	-0.29	0.64	-0.29	0.06
-0.03	0.16	-0.47	0.16	-0.03
0.007	-0.06	0.2	-0.06	0.007

↑ x

↑ N

y →

TABLE (II)

FILTER F2A

Inclination of the Earth's total field:54°

Declination of the Earth's total field with respect to y-axis:90°

Burial depth (depth to the top of the prism):1m

Depth extend:1m

Thickness along N-S direction (X axis):1m

Half strike lenght (y axis):0.5m

Dip of the prism:90°

Number of thin plates used to generate the prism:11

PRODUCED FILTER

↑ x

0	0.001	0.006	0.011	0.006	0.001	0
-0.001	-0.001	-0.003	0.002	-0.003	-0.001	-0.001
-0.0002	-0.0025	0.006	0.025	0.006	-0.0025	-0.0002
-0.01	0.041	-0.17	0.38	-0.17	0.041	-0.01
0.0003	-0.014	0.089	-0.298	0.089	-0.014	0.0003
0.0001	0.0024	-0.043	0.166	-0.043	0.0024	0.0001
-0.0016	-0.0005	0.0156	-0.0709	0.0156	-0.0005	-0.0016

y →

↑ N

TABLE (III)

FILTER F3A

Inclination of the Earth's total field:54°

Declination of the Earth's total field with respect to y-axis:90°

Burial depth (depth to the top of the prism):2m

Depth extend:1m

Thickness along N-S direction (X axis):1m

Half strike lenght (y axis):0.5m

Dip of the prism:90°

Number of thin plates used to generate the prism:11

PRODUCED FILTER

↑ x

0.01	-0.03	0.03	0.09	0.03	-0.03	0.01
-0.05	0.09	-0.12	-0.05	-0.20	0.09	-0.05
0.19	-0.54	1.02	-1.02	1.02	-0.54	0.19
-0.06	2.12	-4.74	6.79	-4.74	2.12	-0.60
0.51	-1.82	4.23	-6.47	4.23	-1.82	0.51
-0.29	1.04	-2.49	3.88	-2.49	1.04	-0.29
0.11	-0.42	0.98	-1.49	0.28	-0.42	0.11

↑ N

y →

TABLE (IV)

FILTER F2A

Inclination of the Earth's total field:54°

Declination of the Earth's total field with respect to y-axis:90°

Burial depth (depth to the top of the prism):1m

Depth extend:1m

Thickness along N-S direction (X axis):2m

Half strike lenght (y axis):0.4m

Dip of the prism:90°

Number of thin plates used to generate the prism:13

PRODUCED FILTER

↑ x

-0.001	0.002	0.011	0.003	0.011	0.002	-0.001
0.003	-0.011	0.021	-0.037	0.021	-0.011	0.003
0.009	-0.026	0.032	0.024	0.032	-0.026	0.009
0.019	-0.040	0.015	0.072	0.015	-0.040	0.019
0.026	-0.034	-0.021	0.022	-0.021	-0.034	0.026
0.023	-0.018	-0.016	0.022	-0.016	-0.018	0.023
-0.003	0.032	-0.060	0.074	-0.060	0.032	-0.003

↑ N

y →

The system of equations (12) was solved using the Gauss-Jordan method in order to confront to the general simplicity for the whole procedure. However, if filters of relatively large length in both directions are attempted to be computed, equations (12) become singular. Thus, if one wishes to produce by his own large filters, is better to use another method to overcome this problem.

Figure (3) and (4) shows the 2 dimensional convolution of filters F2 and F2A with the respective shape functions which were inverted to produce these filters. Both figures provide a test on the whole procedure. Furthermore, they give a check on the performance of the filters. As it was expected, the filter F2A (7X7 positions) which is longer in both directions than the F2 (5X5 positions) functions better. The result is much more pronounced for F2A, less noisy, and more sharp. The last factor gives to the filter an approach to the desired response. In other words, the response of F3A comes closer to the surface projection of the disturbing body.

APPLICATION ON SYNTHETIC DATA

In order to check the performance and applicability of the presented filters, two artificially constructed data sets were employed. In both cases, we used models which simulate structures which often occur in the archaeological search. The total magnetic field effect was calculated using the algorithm given by Bhattacharrya (1964).

Figure (5a) shows the response after convolution of filter F2 with the anomaly produced by a thin rectangular slab buried at 1m depth. The slab is considered positively magnetized with respect to the envinronment possessing a susceptibity contrast of 0.0005

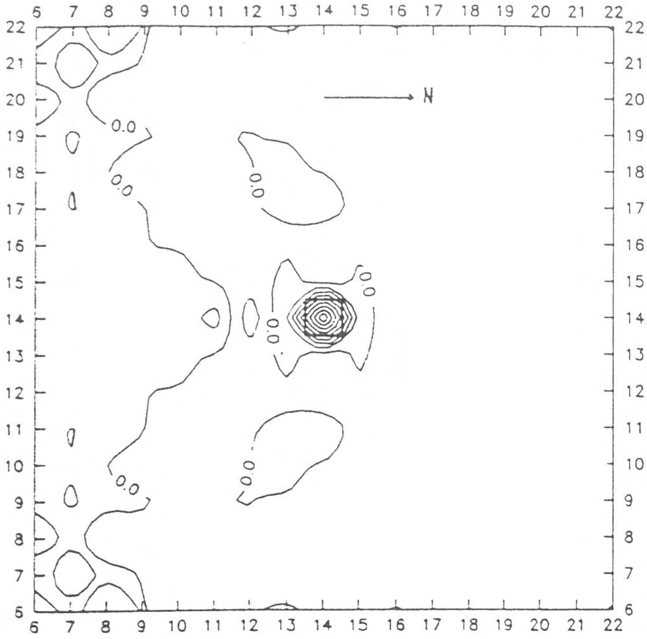

Fig.3. The results of convolution of filter F2 with the respective shape function

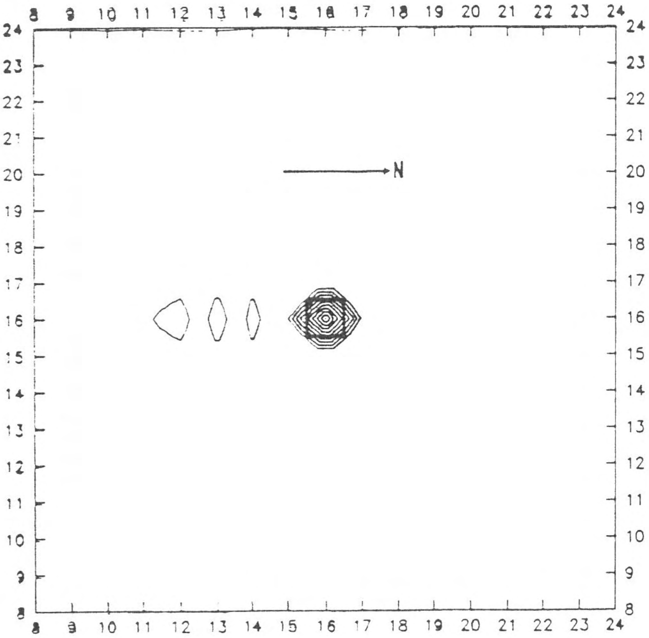

Fig.4. The results of convolution of filter F2A with the respective shape function.

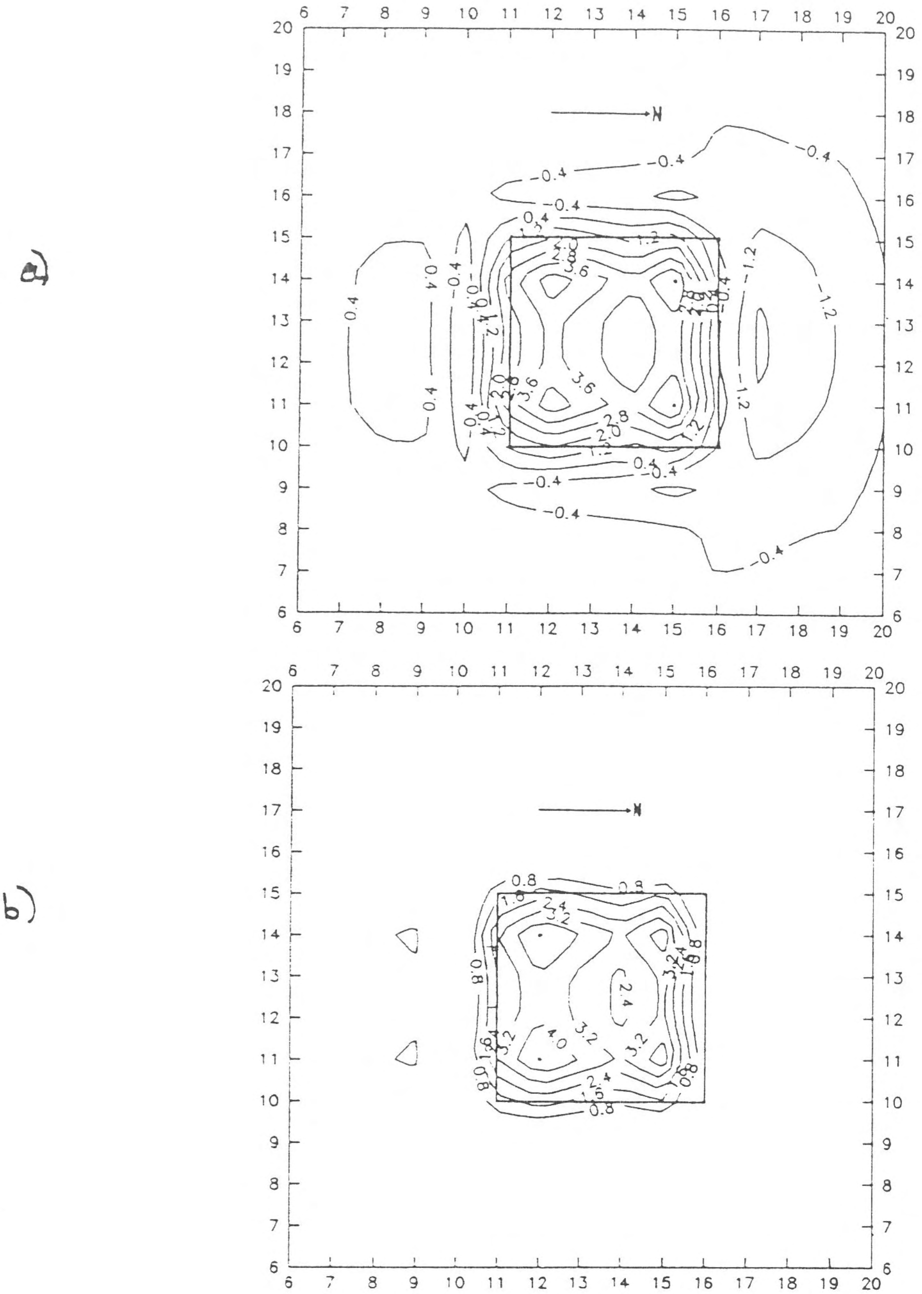

Fig.5. a) The result of convolution of filter F2 with a produced total magnetic field anomaly using a horizontal thin slab model. The plan view of the model is shown with solid line. The susceptibility contrast was consider to be +0.0005 (C.G.S.). b) The same map as in a) after cut-off of the lower contour levels.

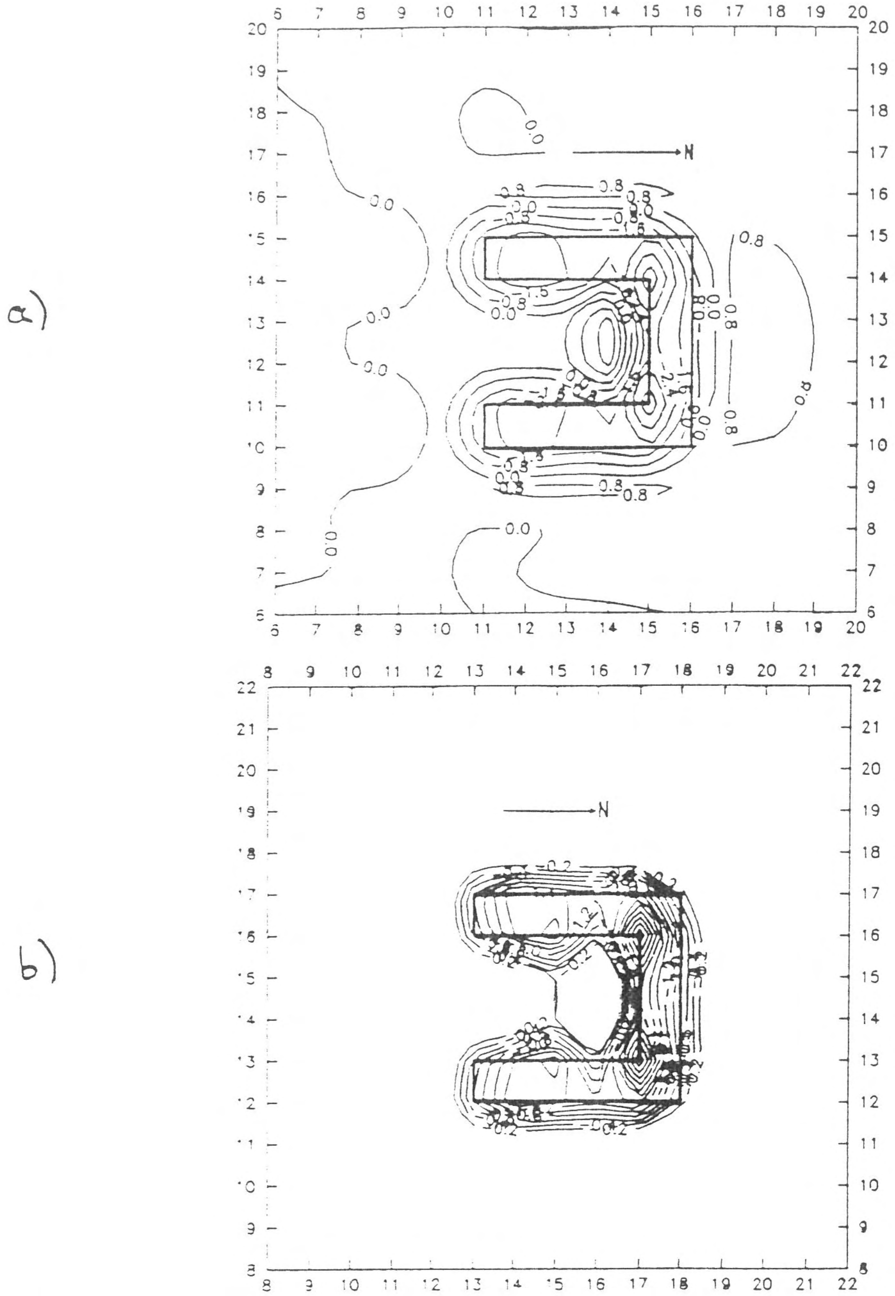

Fig.6. a) The result of convolution of filter F2 with a calculated total magnetic field anomaly using the model whose plan view is shown by solid line. The susceptibility contrast was considered to be +0.0005 (C.G.S.). b) The same map as in a) after cut-off of the higher contour levels.

(C.G.S.). It has a depth extent of 1m and a crossection of 5X5m. The result can be further enhanced if we cut off the lower contours up to some level (figure 5b).

The result of convolution of F2 with the total field anomaly produced by a model which represents the ruins of the foundation of a building are shown in figure (6a). The ruins supposed to be composed of walls of 5m length and 1m wide buried at 1m depth. The susceptibility contrast supposed to be -0.0005 (C.G.S.). The cut-off of the higher contour levels provides the map of figure (6b) which is a rather satisfactory result.

APPLICATION ON THE SITE OF EUROPOS (N.GREECE)

Figure (7) shows the total magnetic field map over a location at the acropolis of the ancient city Europos in N.Greece. Europos was an ancient commercial center byside of the river Axios. The city was mainly flourished at the classical era.

The map of figure (7) presents the residual total field after the subtraction of a 1st degree regional. The data was collected using a proton-precession magnetometer at traverses 1m apart each from the other, stepwise at 1m intervals. Another magnetometer of the same type served as base station in order to reduce the values for the daily variation of the earth's magnetic field.

Figure (8) shows the result after convolution of the total field values with the filter F2A. The positive values of the distribution of amplitude function (fig.8.) have been plotted on figure (9a) while the negative ones on figure (9b). The map of positive values shows very well the location and spatial extent of structures which possess high susceptibility relativelly to the envinronment. In other words, the location of features like "destruction phases", brick walls, ditches, e.t.c. On the

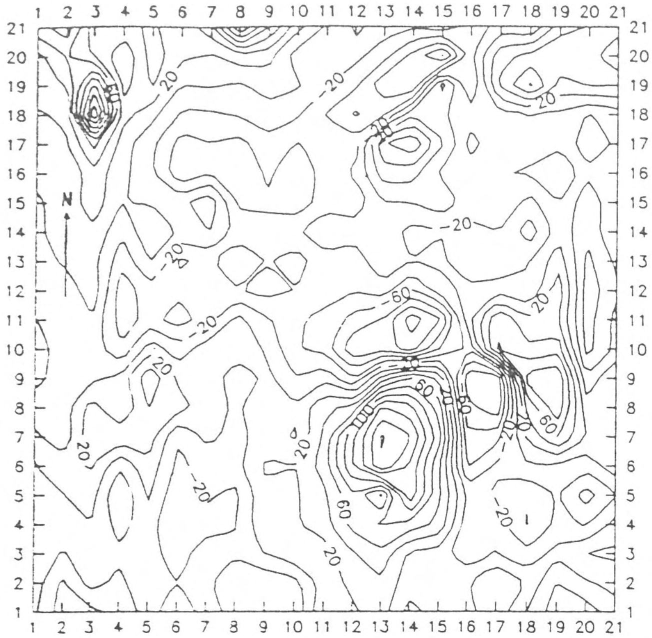

Fig.7. Residual total field map of a subregion of the archaeological site at the acropolis of Europos (N. Greece). Contours are in Gammas.

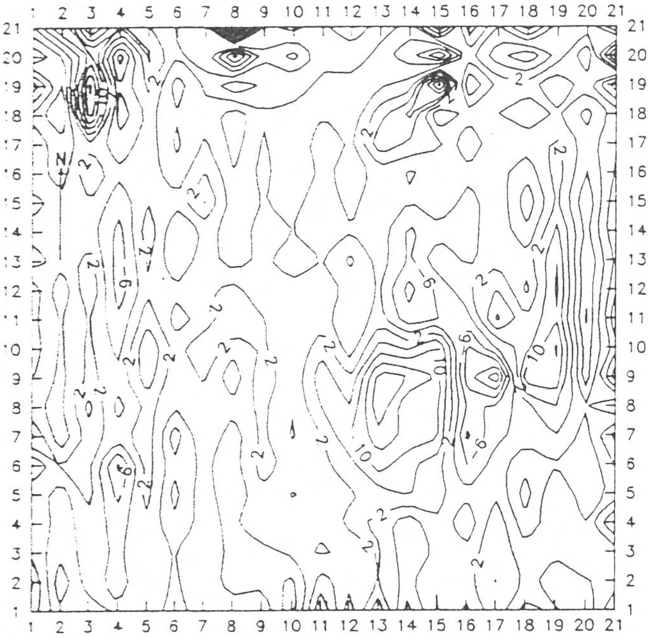

Fig.8. The result of convolution of the filter F2A with the data of figure (7).

contrary, the map of negative values pinpoints structures which possess lower susceptibility than the hosting soil. The later are features like stone built walls, roads made by hewn stones, foundation ruins, e.t.c.

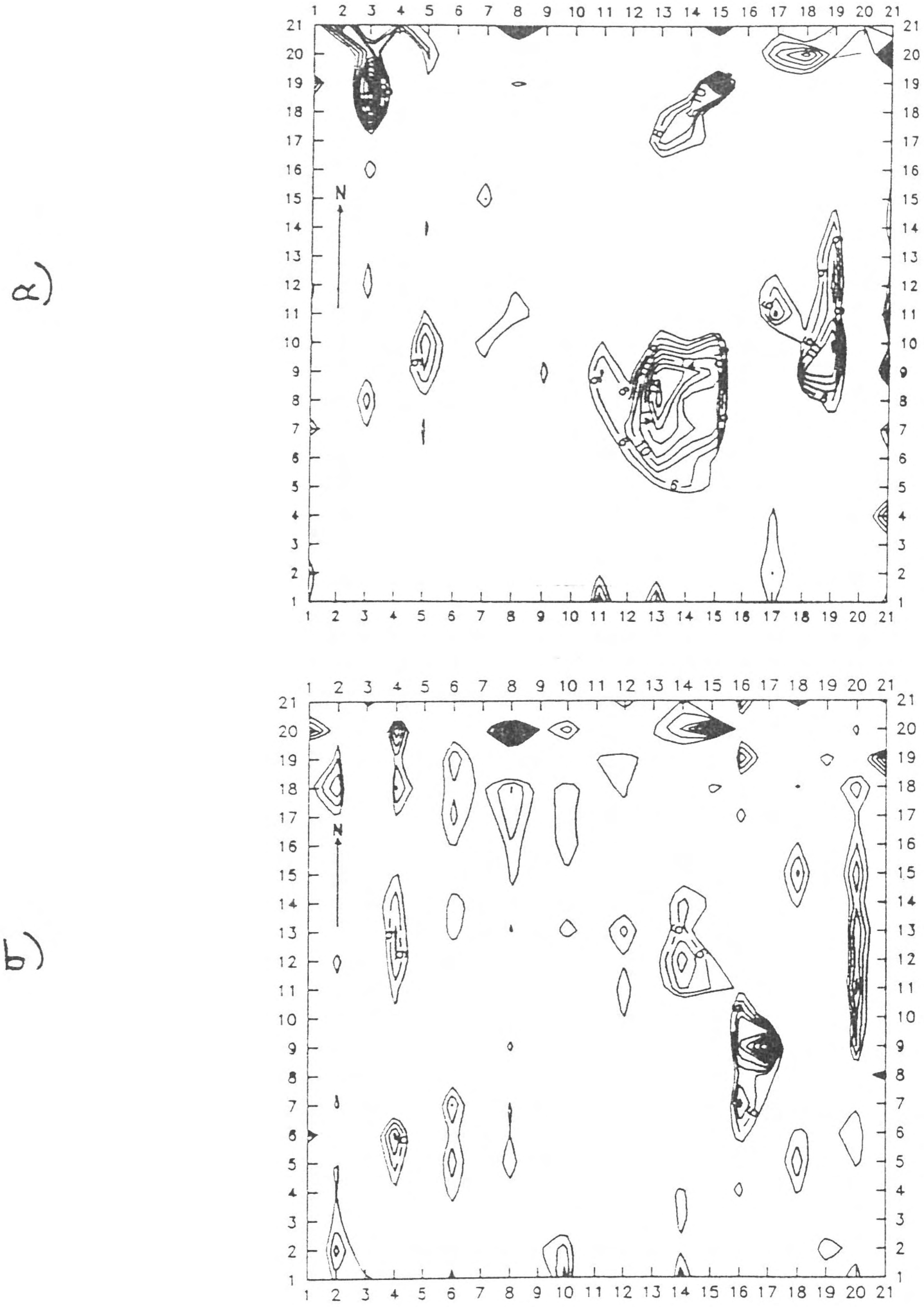

Fig.9. The positive contour levels of figure (8) in a) and the negative ones in b).

DISCUSSION

With respect to the presented examples, either synthetic or actual, it is obvious that the proposed procedure reaches its best performance if the positive and negative values are plotted separately. This is a compromise in order to avoid confusion which might be created if one attempts to interpret the initial amplitude function distribution maps. Alternatively, one may hatch the areas of positive values on the unique map, or present the result as dot-density image. In any case of presentation, the common feature is that the inversion filters function in a manner similar to reduction to the pole. Furthermore, the limits of positive or negative anomalies give a fair approach to the horizontal dimensions of the buried targets.

A crucial point of the whole procedure is the selection of burial depth for the models used to produce the filters. At the presented example of Europos, the burial depth was known to be at about 0.5m below ground surface. The magnetometers sensor was fixed at 0.5m above surface. Hence the overall burial depth was 1m. An attempt to apply the filter F3A which was constructed for 2m burial depth produced noisy results as shown in figure (10). Hence, an estimate of depth level where the antiquities are expected is necessary. This is easily accomplished if a trial pit exists at the site under study. Alternatively, one can use the Historical-Archaeological or any other sort of information about the site.

The overall procedure which was presented in these pages was formulated in a simplified manner as already stated. The simplicity results in increased applicability of the inversion filters without loss of desired information. However, it would be better for anyone who wishes to produce his own filters, to have

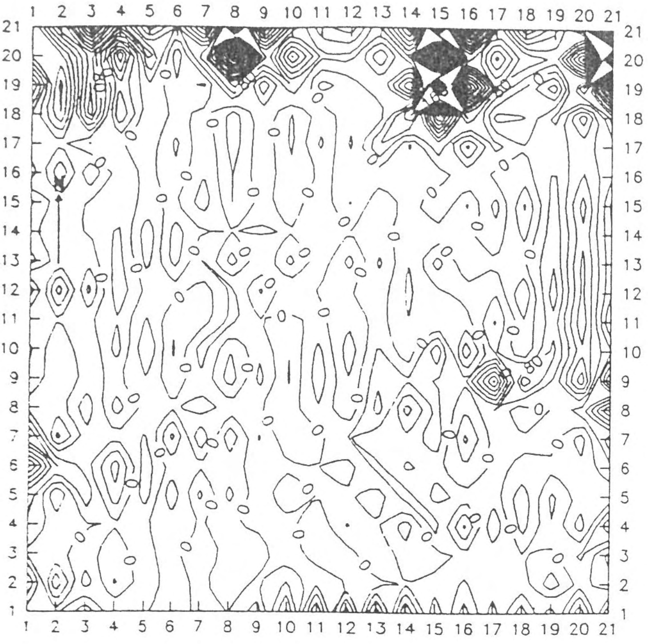

Fig.10. The result of convolution of the filter F3A with the data of figure (7).

them ready and tested on one or two subgrids of the area to be studied. Next, the filters can be applied on large scale at the same area.

Another shortcoming of the simplicity rule is that solution of equation (12) can be achieved by any other method than the Gauss-Jordan used by the authors, resulting thus in more accurate and larger filters.

The question on the appropriate filter length has been posed by Tsokas et al. (1989) who constructed 1-dimensional filters in an attempt to modify the original procedure of Karousova and Karous (1989). The suitable length is that where the autocorrelogram of the shape function (input wavelet) vanishes to zero. In that case compatibility with reflection seismics is achieved (Yilmaz 1986).

However, such a consideration in the present case will result in large filters which are inconvenient for the involved calculations.

The filtration by convolution of the total magnetic field data with the proposed filters is easily applied. This is an advantage of the technique because it can be carried out at the fields by a portable computer yielding thus the results rapidly "in situ".

Inversion filters can be evaluated on the basis of models other than the vertical sided rectangular prism. For instance, vertical or horizontal cylinders simulate some of target structures in the "archaeological geophysics" like kilns, pits, ditches, e.t.c. However, the ensemple of vertical sided prisms can effectively built the majority of models. This aspect is self evident because we are dealing with ruins of past human activity. In addition, it is confirmed by the experience of anyone who is involved with the matter. However, one can follow the described procedure and produce filters for cylidrical structures using an appropriate formula from the literature.

CONCLUSIONS

The proposed procedure of construction of inverse filters and convolution of them with the total magnetic field data seems to function satisfactorily. The results are maps of the spatial distribution of the amplitude function. The anomalies on those maps are located directly over the disturbing structures. The spatial extent of the structures is approximately given by the extent of the anomalies after cutting-off of some levels. The levels which must be removed should have the opposite sign of the susceptibility contrast of the target structure with the envinronment.

The results can be considered as an approximate image of the plan view of the relics. This fact contributes into rapid location and identification of excavational targets.

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